Parametrized design of cylindrical gears with helical teeth

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Abstract. By knowing the importance of gears in industry this paper presents the parameterized design for helical gears. Firstly, it is presented the mathematical equations for generating involute profile of tooth and after for different types of modules and number of teeth. Using the parameterized equations was made the chart for helix. The points are exported in a matrix that will be used to create the sketch of the guide curve of the tooth. This data are imported in a CAD application and with specific commands is realised the 3D model of one gear and after the helical gears. This work has economic benefits because it reduces the time and cost for design and also the cost of the product.

1. Introduction

Mechanical transmission is a system of kinematic elements that has the role of transmitting movement and mechanical energy [1]. The motion is transmitted directly or indirectly, from the driving shaft to the driven shaft. If the distance between the axes of the two shafts is not large, direct transmission can be used with: friction wheels, gear wheels, or screw-nut. In the case of large distances between the axles of the driving and driven shafts, indirect transmission can be used with: chains, belts, cables, or levers.

A gear is a part of a machine intended to set in motion another gear, or to be set in motion by it, by the action of teeth successively and continuously in contact, in order to transmit the moment.

Gear wheels have a wide field of use, they are used in motor vehicle gearboxes, in machine tool gearboxes, in the kinematic chains of machine tools, in various agricultural machines (combines, discs, etc.), in special installations – mill, woodworking machinery, in the food industry, in the textile industry, watches, reducers, etc.[2]

According to [3], the current state of research on parameterized design is represented by the work of scientists from all over the world, so the work presents how database applied in UG secondary development in order to complete the parametric design of helical gear and a library was created to help design. In article [4] is described a new circular arc helical gear, different from conventional arc gears and also there are presented parametric equations for contact curves and another parameters. Article [5] refers at mathematical model and contact analysis of the helical gears and the intersecting lines between profiles were solved by Gauss-Newton method. The object of work [6] presents the development of soft “HelOSpur 2011” which offers to designer the analytical way of calculating needed parameters for design spur and helical gears, and this can make preciously 2D and 3D drawings for gears. The researchers [7] propose a parameterized approach to realise a high-precision 3D model of involute helical gears and the computer cand make too the FEM analysis for contact region. Scientists [8] have been used CATIA for design method of cylindrical spur or helical gears to create 3D model and also the possible profile displacements or FEM analysis for this machine parts.

The parameterized design aims to experiment in a fast way and with low costs different dimensional variants, thus reducing the design time and obtaining the execution documentation.

The problem is to design a cylindrical gear wheel with helical teeth with a module, number of teeth and an angle of inclination of any helix, which, by using a special parameterization program, the dimensions of the wheel can be changed according to the requirements.

Cylindrical gears have an involute tooth profile. The paper aims at the first stage to present the generation of the involute profile of the teeth starting from the parametric equations of the involute, and then the generation of the helix and the realization of the 3D model of a wheel.

2. Generating curve (tooth profile)

The involute is used as the profile of the tooth. According to [9], [10] the involute is a curve that depends on another curve. The involute of a curve is the geometric locus of a fixed point on a straight line that rolls without slipping on the initial (evolved) curve [11]. The involute belongs to a class of curves that is part of the roulette curve family. Due to its properties (the conjugate of the involute is still an involute, from a technological point of view it can be made with simple tools, it allows easy checking of the profile) the involute is used in the construction of gears as a tooth profile [12], [13, [14].

In the case of the circle involute, the circle represents the base; the straight line that rolls without slipping on the base circle represents the roller, and the trajectory (involute) described by some point on the roller represents the roulette (figure 2.1) [15].

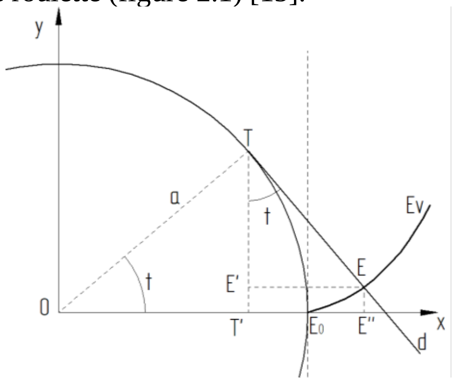

Figure 2.1 Generating the involute.

Notations:

a - radius of the base circle

t - the angle formed by the axis Ox with the radius at the point of tangency of the line rolling over the base circle

d - straight line that rolls without slipping on the circle of radius "a"

E_0 - the initial position of the point on the rolling line

E - some position of the point on the line "d" whose trajectory is being studied.

The parametric equations of the base circle according to figure 2.1 are described by the relations:

$$\begin{cases} x = a \cdot \cos(t) \\ y = a \cdot \sin(t) \end{cases} \tag{2.1}$$

The parametric equations of the involute can be deduced from figure 2.1. The initial position of point M on line "d" rolling without slipping on the base circle is point "A". Since the rolling of the straight line on the circle occurs without sliding, it follows:

$$ET = E_0T = a \cdot t \tag{2.2}$$

$$\begin{cases} x_E = OT' + T'E'' \\ y_E = TT' - TE' \end{cases} \tag{2.3}$$

From figure 2.1 it is noted that:

$$\begin{aligned} OT' &= OT \cdot \cos(t) \\ T'E'' &= TE \cdot \sin(t) \\ TT' &= OT \cdot \sin(t) \\ TE' &= TE \cdot \cos(t) \end{aligned} \tag{2.4}$$

By substitution it follows:

$$\begin{aligned}
&OT' = a \cdot \cos(t)\\
&T'E'' = a \cdot t \cdot \sin(t)\\
&TT' = a \cdot \sin(t)\\
&TE' = a \cdot t \cdot \cos(t)
\end{aligned} \tag{2.5}$$

After replacing (2.3) in (2.5) and performing the calculations, the parametric equations of the involute described by point E are obtained:

$$\begin{cases} x_E = a \cdot \cos(t) + a \cdot t \cdot \sin(t) \\ y_E = a \cdot \sin(t) - a \cdot t \cdot \cos(t) \end{cases} \tag{2.6}$$

or:

$$\begin{cases} x_E = a(\cdot \cos(t) + t \cdot \sin(t)) \\ y_E = a(\sin(t) - t \cdot \cos(t)) \end{cases} \tag{2.7}$$

A more general expression is given by the relation:

$$\begin{cases} x_E = a[\cdot \cos(t) + (t - q) \cdot \sin(t)] \\ y_E = a[\sin(t) - (t - q) \cdot \cos(t)] \end{cases} \tag{2.8}$$

where the term "q" is the angle that defines the starting point of the involute on the circle (figure 2.2).

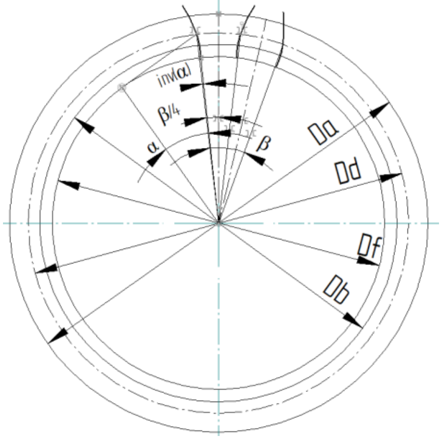

Figure 2.2. Plotting the involute profile.

The angle *q* is determined with the relation:

$$q = \frac{\beta}{4} - \delta \tag{2.9}$$

where β is the angle between two homologous flanks of the toothing; is determined with the relation:

$$\beta = \frac{360}{z} \tag{2.10}$$

where z is the number of teeth on the wheel.

δ is the angle formed by the radius of the dividing circle at the intersection with the involute and the radius of the base circle at the starting point of the involute. It is determined with the relation:

$$\delta = inv(\alpha) = tg(\alpha) - \alpha \tag{2.11}$$

where α represents the pressure angle, α=20o. In the previous relationship, the angle α is entered in radians.
By substitution we get:

$$q = \frac{\beta}{4} - [tg(\alpha) - \alpha] \tag{2.12}$$

With the notations in figure 2.2 the parametric equations of the involute flanks become:

$$\begin{cases} x_E = Rb \cdot \left(\cos(t) + \left(t - \left(\frac{\beta}{4} - (tg(\alpha) - \alpha) \right) \right) \cdot \sin(t) \right) \\ y_E = Rb \cdot \left(\sin(t) - \left(t - \left(\frac{\beta}{4} - (tg(\alpha) - \alpha) \right) \right) \cdot \cos(t) \right) \end{cases} \tag{2.13}$$

where “t” takes values between 0 and 0.7

Thus, in figure 2.3 shows the profile of the teeth in normal section for m=3 and z=20, in figure 2.4 shows the profile of the teeth in normal section for m=2 z=40, in figure 2.5 shows the profile of the teeth in normal section for m=3 z=20, and in figure 2.6 shows the tooth profile in normal section for m=2 z=20.

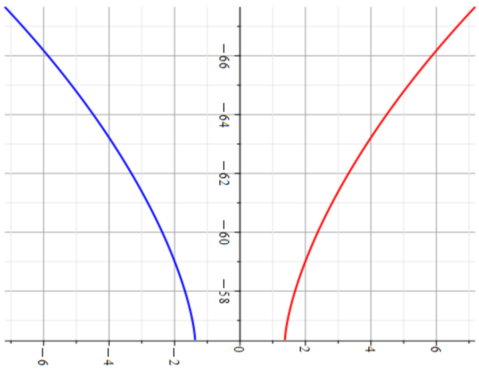

Figure 2.3. Tooth profile in normal section for m=3 z=40

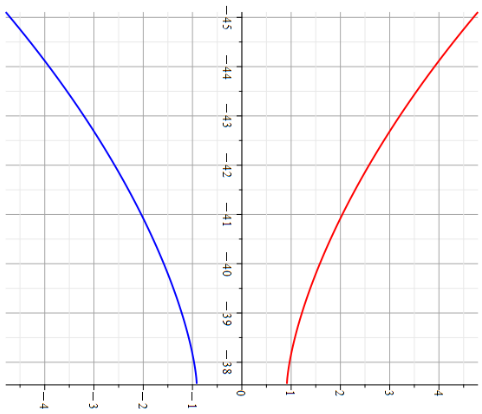

Figure 2.4. Tooth profile in normal section for m=2 z=40

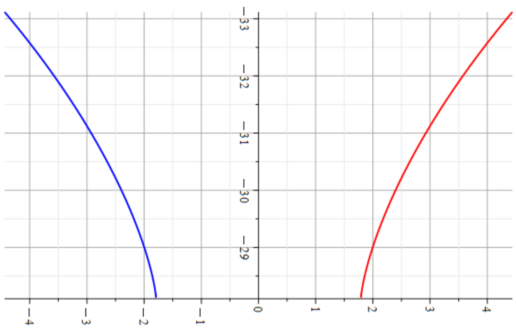

Figure 2.5. Tooth profile in normal section for m=3 z=20

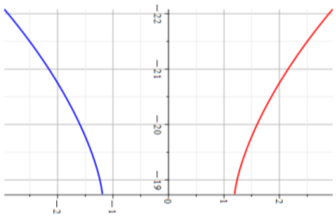

Figure 2.6. Tooth profile in normal section for m=2 z=20

3. 3D model of helical gear

It is considered a gear with transmission ratio i=1, consisting of cylindrical gears with helical teeth with the following characteristics:

Module m= 3 mm

No. of teeth z=30

The inclination angle of the teeth on the division circle β=30º.

Based on the initial elements, knowing the calculation relations for the geometric parameters indicated in [15], the helix pitch is determined.

$$p = \frac{Dd \cdot \pi}{\tan(\beta)}. \tag{3.1}$$

where Dd is the dividing diameter.

The parametric equations of the helix are:

$$\begin{cases} x = \dfrac{D_d}{2} \cdot \cos(\varphi) \\ y = \dfrac{D_d}{2} \cdot \sin(\varphi) \\ z = \dfrac{\varphi \cdot p}{2 \cdot \pi} \end{cases} \tag{3.2}$$

where "φ" represents the helix rotation angle from 0 to 2π.

In the figure 3.1 is presented the helix chart, and similarly to what was presented previously, the helix points are exported in a matrix (figure 3.2) that will be used to create the sketch of the guide curve of the tooth.

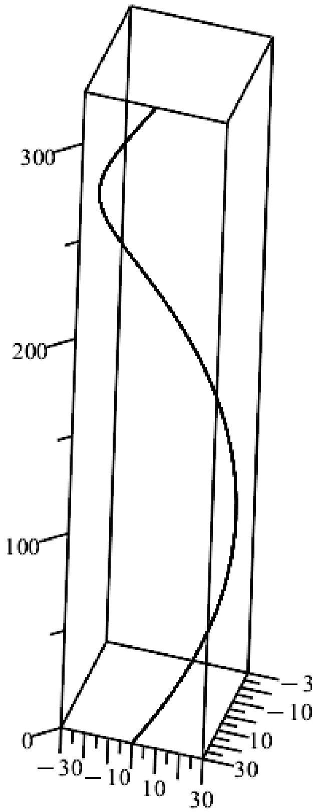

Figure 3.1. Helix (directory) of the tooth of the gear wheel

coordonate elice dinte		
x	y	z
30	0	0
29.63065022	4.693033951	8.162097139
28.53169549	9.270509831	16.32419428
26.73019573	13.61971499	24.48629142
24.27050983	17.63355757	32.64838856
21.21320344	21.21320344	40.8104857
17.63355757	24.27050983	48.97258283
13.61971499	26.73019573	57.13467997
9.270509831	28.53169549	65.29677711
4.693033951	29.63065022	73.45887425
1.83772E-15	30	81.62097139
-4.693033951	29.63065022	89.78306853
-9.270509831	28.53169549	97.94516567
-13.61971499	26.73019573	106.1072628
-17.63355757	24.27050983	114.2693599
-21.21320344	21.21320344	122.4314571
-24.27050983	17.63355757	130.5935542
-26.73019573	13.61971499	138.7556514
-28.53169549	9.270509831	146.9177485
-29.63065022	4.693033951	155.0798456
-30	3.67545E-15	163.2419428
-29.63065022	-4.693033951	171.4040399
-28.53169549	-9.270509831	179.5661371
-26.73019573	-13.61971499	187.7282342
-24.27050983	-17.63355757	195.8903313
-21.21320344	-21.21320344	204.0524285
-17.63355757	-24.27050983	212.2145256
-13.61971499	-26.73019573	220.3766228
-9.270509831	-28.53169549	228.5387199
-4.693033951	-29.63065022	236.700817
-5.51317E-15	-30	244.8629142
4.693033951	-29.63065022	253.0250113
9.270509831	-28.53169549	261.1871084
13.61971499	-26.73019573	269.3492056
17.63355757	-24.27050983	277.5113027
21.21320344	-21.21320344	285.6733999
24.27050983	-17.63355757	293.835497

Figure 3.2 Matrix with the coordinates of the points belonging to the helix (director) of the wheel flanks

Figure 3.3 shows the guide curve (helix) and the closed contour formed by the involutes of the flanks imported into the 3D modeling application, then the 3D model of the wheel body is created (figure 3.4)

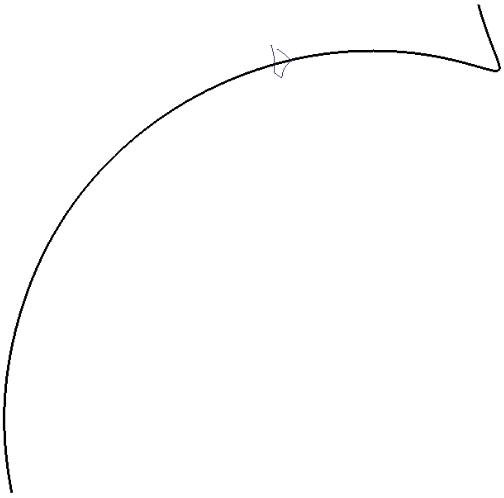

Figure 3.3 The helix (guide curve) and involutes (generators) of the flanks

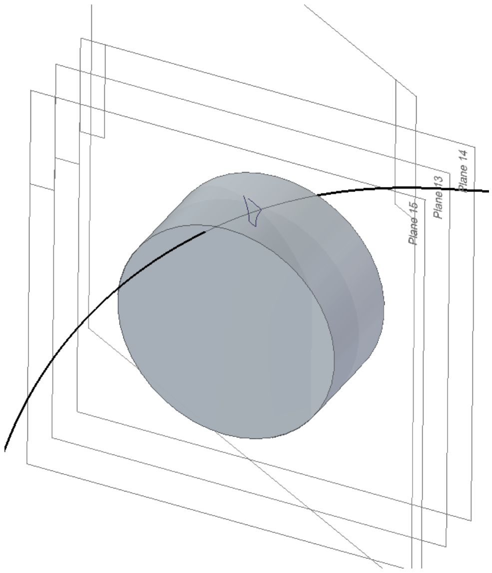

Figure 3.4. 3D model of the wheel body

Using cutting operations following a certain contour, the material corresponding to the gap between two teeth is removed (figure 3.5) and proceeding similarly to what was presented previously, the 3D model of the cylindrical wheel with helical teeth was obtained (figure 3.6).

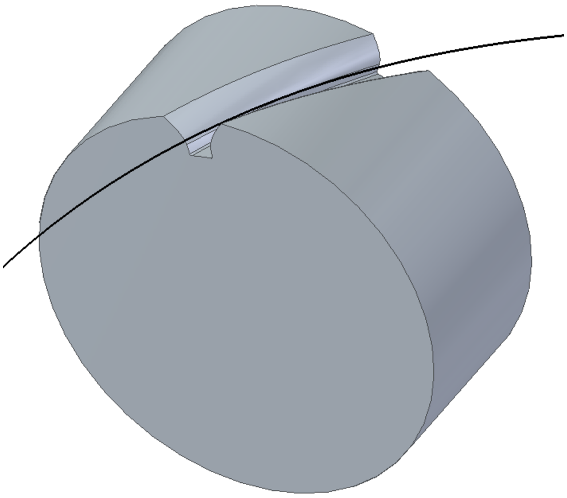

Figure 3.5. Making the gap between two teeth

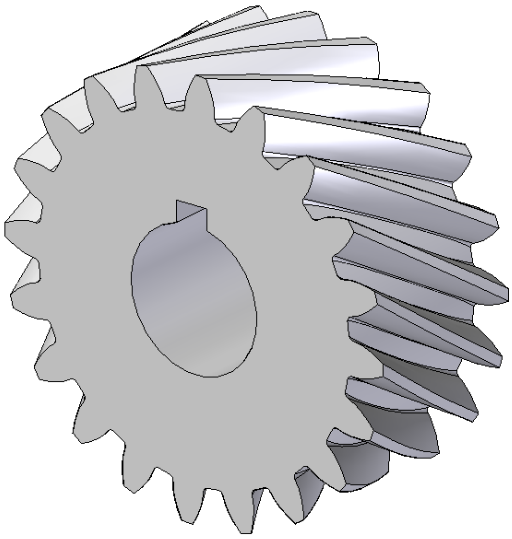

Figure 3.6. 3D model of spur gear with helical teeth

The gear formed with cylindrical gears with helical teeth is shown in figure 3.7.

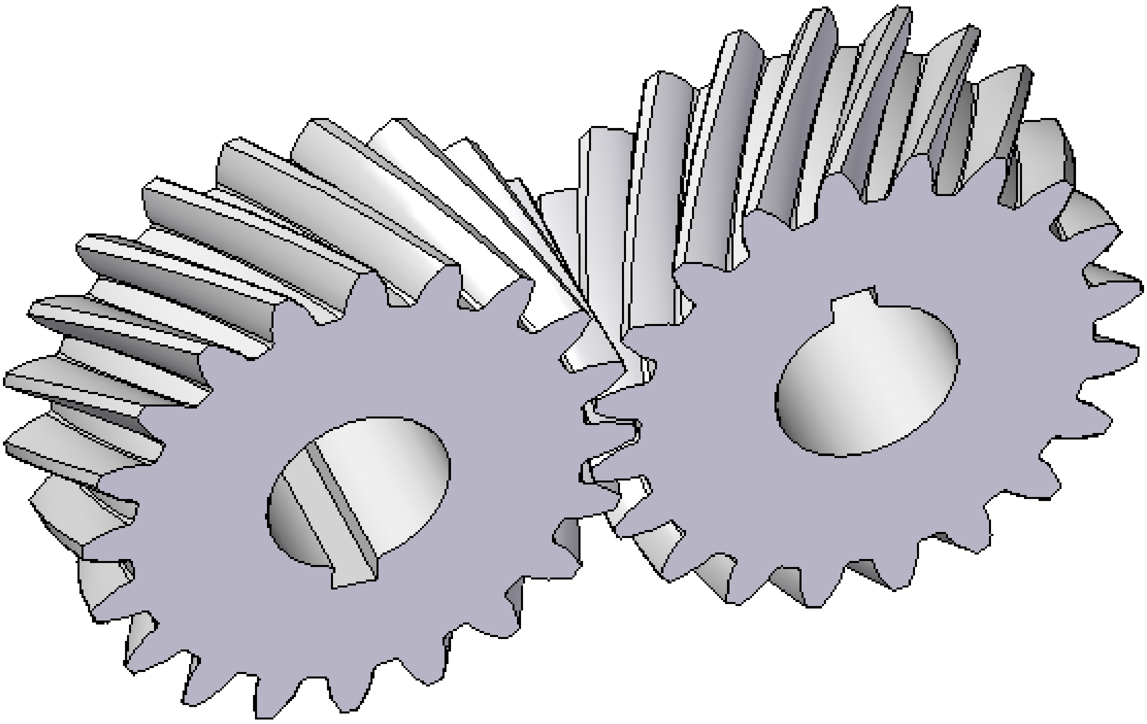

Figure3.7 Gearing with cylindrical gears with helical teeth.

4. Conclusions

The paper presents the parametric equations of the generation of the involute profile of the teeth and the parametric equations of the helix of the teeth. Points were determined with a calculation program, created for this purpose. These are used to determine the points that make up the curves that are used within a CAD application to create the 3D model.

With the help of a calculation program, the parameterized 3D model was created. The calculated program has the advantage of reducing the design time and allows generating the technical documentation for the gears in a shorter time frame, with high precision, thus both reducing the costs related to the design and the manufacturing time of the the wheels.

5. References

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